

Analytical Modeling of Magnetic Fields and Winding Factors for Coreless Stator Axial Flux PM Machines with Different Coil Arrangements

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Abstract—The theoretical framework described in the paper is developed for the analytical modeling of coreless-stator axial-flux permanent-magnet (AFPM) machines to enable informed preliminary design choices, particularly for selecting coil/pole combinations, and fast calculations with satisfactory accuracy. A new generalized winding-factor theory applicable to typical and most frequent coreless AFPM motor topologies employing a double-sided PM rotor and any winding configuration is formulated, and detailed calculations with example results are provided for typical air-gap winding layouts. The analysis is complemented by comprehensive discussions on innovative winding types, power loss mitigation, design and manufacturing flexibility, fault tolerance, and reliability aspects. Analytical expressions for the air-gap flux density of double-sided surface-mounted permanent magnet (SPM) rotors, torque, and back electromotive force (B-EMF) are derived. The accuracy of the proposed analytical approach is validated against 3D finite element analysis (FEA) and experimental measurements from two printed-circuit-board (PCB) stator AFPM prototypes with different pole numbers and winding types. The findings support rapid evaluation of critical design choices, such as coil/pole combinations and pole count, and provide a computationally efficient approach for the initial stages of design sizing and optimization of coreless-stator AFPM machines.

Index Terms—Analytical, axial flux permanent magnet machine, coil-pole combination, coreless stator, electromagnetic finite element analysis, PCB stator, permanent magnet synchronous motor, winding factor.

I. INTRODUCTION

Coreless stator axial-flux permanent-magnet (AFPM) machines have attracted significant attention from both industry and academia in recent years, with research publications indicating an approximately 100% increase between 2020 and 2024 compared with the preceding period [1]. In this class of axial-flux machines, the stator lacks ferromagnetic materials

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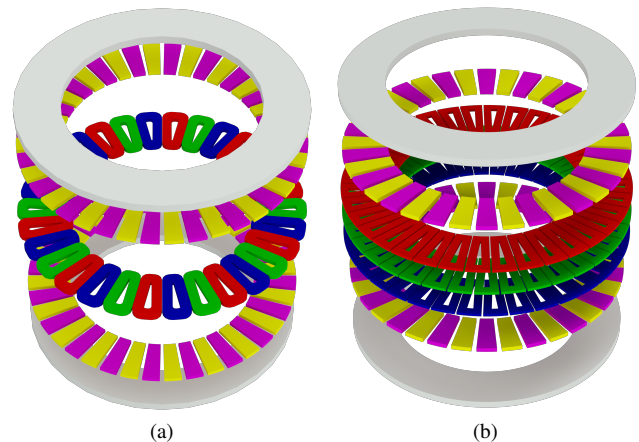


Fig. 1. Coreless stator axial flux permanent-magnet machines with double-sided surface-mounted PM rotors and (a) single-layer winding with a 3-coil/4-pole configuration, and (b) three-layer winding with the identical pole and coil count in which each layer corresponds to one phase.

and is composed solely of conductors, and the rotor employs permanent magnets (PMs), typically in surface-mounted configurations or Halbach arrays for high-power applications. The structures of two representative designs, featuring single-layer and three-layer winding configurations, are shown in Fig. 1.

Due to coreless configuration, core losses are eliminated, enabling potentially higher efficiency and power density [2]–[5]. These machines feature a compact pancake-shaped geometry and exhibit inherently low acoustic noise due to the absence of saliency and reluctance torque [6]–[8]. These features, together with the growing maturity of their design and manufacturing processes, make them strong candidates for a wide range of applications, spanning from relatively low-power heating, ventilation, and air-conditioning (HVAC) systems to high-power-density electric aircraft propulsion and large direct-drive wind turbine generators.

Conventionally, Litz wire has been used in coreless stator machines due to their high copper fill factor. The absence of a stator core enables the use of printed circuit board (PCB) windings instead of Litz-wire windings. Printed circuit board stators usually reach about half the copper fill factor of Litz wire stators [9]. Litz-wire stators are generally expected to

provide higher copper fill and enable a thinner stator than PCB implementations. However, Litz-wire stators typically involve higher material and manufacturing costs than PCB windings.

The PCB stator windings offer significant advantages in production costs and scalability [10], [11], and can be manufactured through fully automated, repeatable, and low-cost processes with high reliability [9], [12], [13]. While securing coils in coreless stators requires additional fixtures because there are no slots, PCB stators inherently overcome this challenge. Printed-circuit-board stators also enable highly compact integration of the motor and drive by incorporating the drive electronics directly onto the same PCB as the windings. For example, the empty space at the center of the coils can be utilized for position sensors, while the non-active regions of the PCB can be used to mount controllers and power electronic drive circuits [14].

Combining these PCB-related advantages with the inherent structural benefits of the coreless topology, such as the elimination of stator laminations, reduced rotor fabrication complexity since the rotor core does not require lamination, and improved accessibility of the windings for more effective cooling, results in a simplified and potentially more cost-effective machine structure. Commercial product pricing further suggests that coreless AFPM machines can be competitive with well-established interior permanent-magnet (IPM) machines for similar output ratings.

The industry has announced products of PCB-stator AFPM machines across a wide range of ratings. For instance, Infinium Electric, Inc. [15] and E-Circuit Motor (ECM), Inc. [16] offer solutions for HVAC and e-mobility systems. They have recently been considered an attractive alternative for cooling and ventilation, especially in data centers, where the growing demand for artificial intelligence (AI) infrastructure necessitates highly efficient, low-noise, and compact cooling solutions. Examples of the application of PCB stator technology across an entirely different power level can be found in the research and development of large direct-drive wind turbine generators, such as those manufactured by Boulder Wind Power, Inc. [17].

Another application of coreless stator AFPM machines is aerospace propulsion, owing to their relatively high specific power density (kW/kg) and strong potential for effective forced cooling [1]. The elevated specific power density arises from inherent features such as the absence of a ferromagnetic stator core, the capability for direct cooling, particularly at very low temperatures, and topology modifications including the use of Halbach-array PM rotors [18]–[20]. These advantages are reflected in commercial developments by LaunchPoint Electric Propulsion Solutions, Inc. [21], [22], as well as in NASA research projects [23], [24].

Axial flux machines are complex 3D electromagnetic problems due to radial flux variations and fringing effects at both outer and inner radii, as noted by [25], [26]. In coreless stator AFPM machines, these issues are exacerbated by the larger air-gap and absence of the stator core. Accurate modeling necessitates 3D FEA calculations, which are computationally

intensive and time-consuming for large-scale design optimizations that require evaluation of multiple coil (slot-equivalent) and pole combinations.

A two-dimensional (2D) modeling approach for AFPM machines was proposed in [27]. In this method, the machine is cut at the average radius to obtain a cylindrical surface, which is then unrolled to form an equivalent 2D representation. In this paper, the equivalent 2D approach is employed to calculate the air-gap flux density and to derive the torque equation.

To facilitate the design optimization process, this paper develops a comprehensive analytical framework for determining suitable pole/coil combinations and performing the initial electromagnetic sizing of the machine. The main contribution of the proposed framework is not to replace 3D FEA. It provides fast and easy-to-implement analytical tools to select critical design variables prior to computationally intensive 3D FEA-based optimization, thereby significantly reducing the number of required simulations.

Prior research in this field does not provide a general analytical tool for the systematic selection of critical design variables in coreless stator AFPM machines. Existing methodologies [28], [29] are topology-dependent and tailored to specific winding configurations and magnet geometries, limiting their broader applicability. Similarly, the winding factor formulation presented in [30] is derived for a single-winding geometry and yields expressions that depend on the pole number, requiring the pole number to be predetermined, and also neglects pitch and group factors. An analytical model that aims to achieve the accuracy of 3D FEA and experimental validation [31] is also restricted to particular motor topologies.

This paper extends previous works in [32]–[34], and makes several contributions. Winding-factor theory and calculation procedures are presented for typical coil/pole combinations, which is also applicable to any general winding configuration. The no-load magnetic flux density expression for double-sided surface-mounted PM rotors is obtained using an equivalent 2D representation of the axial-flux machines (Fig. 2). Analytical equations for torque and back-EMF are derived and validated using 3D FEA and experimental measurements.

Essential design considerations for coil/pole selection, including flexibility and reliability, are discussed. The paper presents a high-fidelity analytical model that can reduce reliance on 3D FEA. The developed framework establishes a systematic and topology-independent analytical foundation for selecting pole/coil combinations in coreless-stator AFPM machines, applicable across diverse applications, geometric envelopes, power ratings, and stator technologies, including PCB and Litz-wire windings.

II. WINDING FACTOR THEORY

The winding-factor formulation developed in this work is valid for the typical and most frequent coreless AFPM motor topologies employing a double-sided PM rotor, and any type of winding configuration, in addition to those exemplified in the paper. As will be explained in the discussion section, the method can be applied, at least for approximate results that

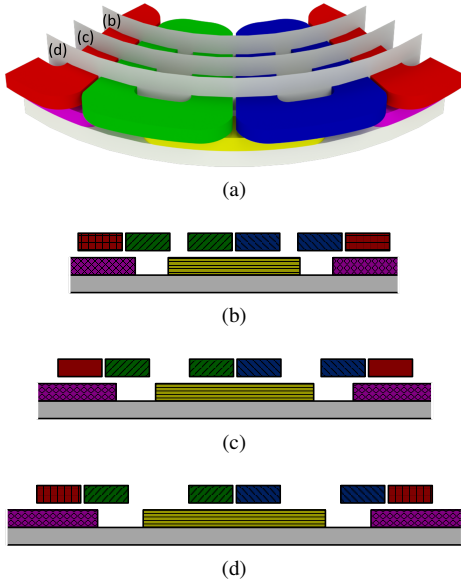


Fig. 2. Equivalent two-dimensional modeling of a coreless AFPM machine, illustrated at three radial positions across different diameters, to demonstrate the curvature effect.

support the initial steps of a design process, to other types of machines with a small stator-to-rotor air-gap.

The winding factors were calculated by considering the effects of distribution and chording within an individual coil. These are influenced by the coil geometry, which depends on the pole/coil combination. For combinations with more than one coil within the smallest geometric periodicity, a coil-group factor must also be included. Consequently, the per-phase winding factor is obtained as the product of the distribution factor, pitch (chording) factor, and coil group factor.

It is assumed that the open-circuit magnetic field established by the rotor is constant, i.e., uniform, along the axial direction through the airgap. This is indeed the case for typical modern designs with high electromagnetic loading, in which case the air-gap and winding thickness are relatively small, as compared with the pole pitch. The uniformity of the field is illustrated in Fig. 3, supporting the validity of the assumption. In this case, in order to greatly simplify the analysis to a single dimension, i.e., along the unrolled circumferential (tangential) direction, the conductors are modeled, as per the typical approach, as infinitely thin and placed in the middle of the airgap.

The proposed approach is somewhat similar to the conventional theory for machines with a ferromagnetic core, for which the conductors are modeled as infinitely thin on the stator surface facing the airgap, and the stator core is considered unsaturated, such that magnetic flux is perpendicular to the surface, mathematically corresponding to a Dirichlet boundary condition. For our coreless double-sided rotor machines, an equivalent Dirichlet boundary condition is valid in the middle of the air-gap due to symmetry.

Furthermore, the winding can be divided into multiple layers along the axial direction, each of the layers having in the center

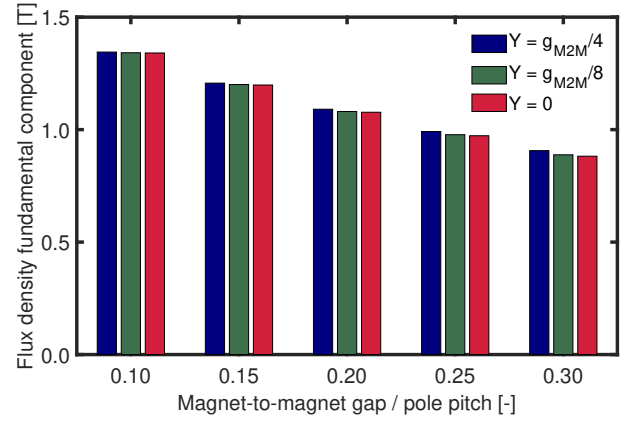


Fig. 3. Normal flux density fundamental components at different positions in the magnet-to-magnet (M2M) gap and at different distances from the magnet surfaces, where $y = 0$ corresponds to the center of the M2M gap, and results are shown for different M2M gap sizes.

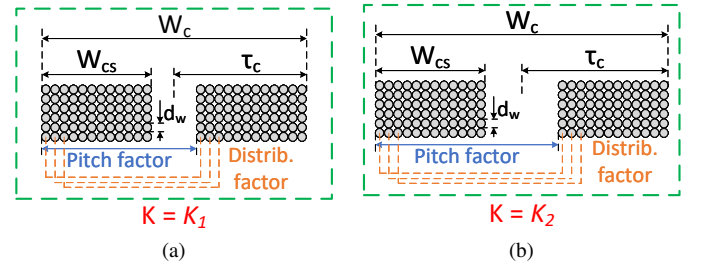


Fig. 4. Radial cut-out of a coil at two radii: (a) smaller radius and (b) larger radius, highlighting the impact of radius on the conductor distribution and pitch factors.

infinitely thin conductors. The axial variations of the magnetic field are considered through a two-dimensional equation of the rotor flux density and by integrating, i.e., summing, contributions from the equivalent layers. In any case, it should be kept in mind that, as explained, the objective of the proposed analytical model is to support the initial design, such as the selection of winding type, coil/pole combinations etc, and not to completely replace more precise electromagnetic 3D FEA, which can be employed, as needed, in the final stages of the design process, resulting all together in reduction of computational efforts.

A. Conductor Distribution

In a coreless stator winding, the conductors are directly exposed to the rotor's magnetic field, leading to non-uniform flux linkage among the individual turns within a coil. Each conductor experiences a different flux linkage angle; thus, the total flux linkage of the coil is the vector summation of the contributions from all the conductors. This concept is captured through the definition of the conductor distribution factor. In coreless stator axial flux machines, the distribution of conductors varies with radius due to the curvature effect, as illustrated in Figure 4 for two representative radial positions.

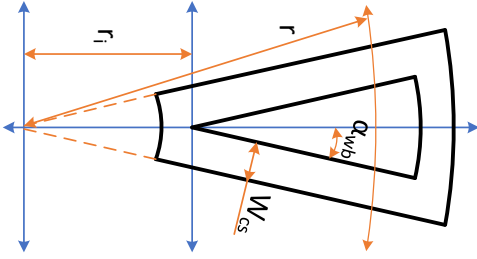


Fig. 5. Two-dimensional top view of a coil with defined geometric variables used in the development of the conductor winding factor equation.

A single general machine winding consisting of a group of coils connected in series is equivalent to a series connection of an ideal sinusoidally distributed winding harmonically related in space distribution. The conductor distribution can then be expressed as a Fourier expansion with a zero average term:

$$N = \sum_{\nu=1}^{\infty} N_{\nu} \cos(\nu\theta + \varphi_{\nu}), \quad (1)$$

where N_{ν} is the amplitude of the ν^{th} harmonic, θ denotes the angular position, and φ_{ν} is the phase angle.

In coreless stator axial flux machines, the coil span increases from the inner radius to the outer radius due to the inherent curvature of the geometry. In contrast, the physical width of each coil side, which contains a fixed number of turns, remains constant along the radial direction. As a result, the corresponding angular width of the conductors changes with radius, causing the conductor distribution to vary radially. Therefore, the Fourier expansion of the conductor distribution must be formulated as a radius-dependent function, which can be expressed as follows:

$$\begin{aligned} N &= \sum_{\nu=1}^{\infty} N_{\nu} \cos\left(\nu \frac{x}{r} + \frac{x_{\nu}}{r}\right) \\ &= \frac{1}{2} \sum_{\nu=1}^{\infty} N_{\nu} e^{j\left(\frac{\nu x}{r} + \frac{x_{\nu}}{r}\right)} + N_{\nu} e^{-j\left(\frac{\nu x}{r} + \frac{x_{\nu}}{r}\right)} \quad (2) \\ &= \frac{1}{2} \sum_{\nu=1}^{\infty} \overline{N}_{\nu} e^{j\left(\frac{\nu x}{r}\right)} + \overline{N}_{\nu} e^{-j\left(\frac{\nu x}{r}\right)}, \end{aligned}$$

where:

$$\overline{N}_{\nu} = N_{\nu} e^{j\left(\frac{x_{\nu}}{r}\right)} = \frac{2}{T_M} \int_0^{\frac{2\pi}{T_M}} F(x) e^{-j\left(\frac{\nu x}{r}\right)} dx, \quad (3)$$

x corresponds to the linear coordinate in the unrolled 2D model, and r represents the radial coordinate in the cylindrical model.

Considering a cut along the circumferential direction denoted by x for the smallest electromagnetic periodicity region and the coil side located at W_s with a coil-side span W_{cs} , containing N_s turns. The conductor distribution contributed by this coil side over the periodic width T_M can then be expressed

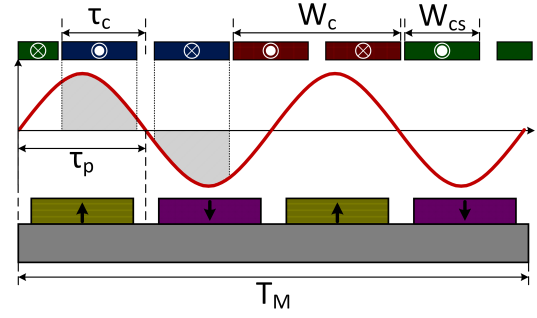


Fig. 6. Two-dimensional visualization of the pitch factors in an example coreless stator winding.

as:

$$\begin{aligned} \overline{N}_t &= \frac{2}{T_M} \int_{\frac{2\pi}{T_M}(W_s - \frac{W_{cs}}{2})}^{\frac{2\pi}{T_M}(W_s + \frac{W_{cs}}{2})} \frac{N_s T_M}{2W_{cs}\pi} e^{-j\left(\frac{2\pi x\nu}{T_M}\right)} dx \\ &= \frac{2N_s}{W_{cs}\pi\nu} e^{-j\left(\frac{2W_s\pi\nu}{T_M}\right)} \sin\left(\frac{W_{cs}\pi\nu}{T_M}\right). \end{aligned} \quad (4)$$

Since W_{cs} equals the product of the number of turns and the conductor width, and because the conductor cross-section is made very small in coreless stator windings to mitigate conductor eddy-current losses, this results in:

$$\begin{aligned} \text{if: } W_{cs} \rightarrow 0 &\Rightarrow \left[\frac{T_M}{W_{cs}\pi\nu} \sin\left(\frac{W_{cs}\pi\nu}{T_M}\right) \right] \rightarrow 1 \\ &\Rightarrow \overline{N}_{t_{max}} = \frac{2N_s}{T_M} e^{-j\left(\frac{2W_s\pi\nu}{T_M}\right)}. \end{aligned} \quad (5)$$

Therefore, the conductor distribution factor is defined as the ratio of the conductor distribution given in (4) to the corresponding maximum achievable distribution in (5), expressed as follows:

$$\begin{aligned} K_T &= \frac{\overline{N}_t}{\overline{N}_{t_{max}}} = \frac{T_M}{W_{cs}\pi\nu} \sin\left(\frac{W_{cs}\pi\nu}{T_M}\right) \\ &\xrightarrow{T_M=2\pi r} K_T = \frac{2r}{W_{cs}\nu} \sin\left(\frac{W_{cs}\nu}{2r}\right), \end{aligned} \quad (6)$$

which can be expressed in a more general form through the following manipulation:

$$W_c = \frac{2\pi r}{N_c} \Rightarrow \tau_c = \frac{\pi r}{N_c} \xrightarrow{\text{if } K = \frac{W_{cs}}{\tau_c}} K_T = \frac{2N_c}{K\pi\nu} \sin\left(\frac{K\pi\nu}{2N_c}\right), \quad (7)$$

where N_c denotes the base coil number for each winding configuration, which is explained in the next section for each winding type. The value of the parameter K varies with the radius at which the distribution factor is calculated. The effect

of the radius on the coil side distribution for an example coreless AFPM machine is illustrated in Fig. 2. As the radius increases, the coil pitch expands while the coil side width remains constant, reducing the K .

Alternatively, (6) can be reformulated based on the coil's inner edge radius, r_i , and the calculation radius, r . Considering the coil geometry illustrated in Fig. 5, K_T can be calculated as:

$$W_{cs} = \tau_c - \frac{W_b}{2} = \frac{\pi r}{N_c} - \frac{\pi(r - r_i)}{N_c} = \frac{\pi r_i}{N_c}$$

$$\Rightarrow K_T = \frac{2N_c r}{\pi r_i \nu} \sin\left(\frac{\pi r_i \nu}{2N_c r}\right). \quad (8)$$

B. Pitch Factor

The pitch factor quantifies the ratio of the induced voltage in a coil to that of a full-pitch coil and is affected by the winding configuration. The representation of the pitch factor is illustrated in Figs. 4 and 6. This factor is also influenced by the curvature effect, as the coil side-to-pitch ratio varies with the radial position. As the coils extend toward larger radii, the distance between the go and return paths increases due to a reduced ratio of coil-side width to coil pitch. This geometric effect leads to a decrease in the induced voltage.

The pitch factor has the same definition and formulation as in conventional cored electrical machines, as for example established in [35]. In conventional machines, the pitch factor can be calculated from:

$$K_p = \cos\left(\frac{\nu\epsilon}{2}\right) = \sin\left(\frac{\nu(\pi - \epsilon)}{2}\right), \quad (9)$$

where ϵ is the short-pitch angle in electrical degrees.

For the coreless motor, the formulation remains conceptually identical. Because the conductors are not positioned in slots, the pitch angle must be defined based on the actual span of the corresponding conductors, $W_c - W_{cs}$, which is illustrated in Fig. 4. Therefore, the effective span of the conductors in electrical degrees can be written as:

$$\pi - \epsilon = \frac{2\pi(W_c - W_{cs})}{T_M}. \quad (10)$$

By substituting (10) into (9), the pitch factor for coreless stator axial flux machines can be expressed as:

$$K_p = \sin\left(\frac{\nu\pi(W_c - W_{cs})}{T_M}\right) \xrightarrow{W_c = \frac{T_M}{N_c}} K_p =$$

$$\begin{cases} \sin\left(\frac{\nu\pi(2 - K)}{2N_c}\right) & , \text{ for } W_{cs} = \frac{KW_c}{2} \\ \sin\left(\frac{\nu\pi(2r - r_i)}{2rN_c}\right) & , \text{ for } K = \frac{W_{cs}}{\tau_c} = \frac{r_i}{r} \end{cases} \quad (11)$$

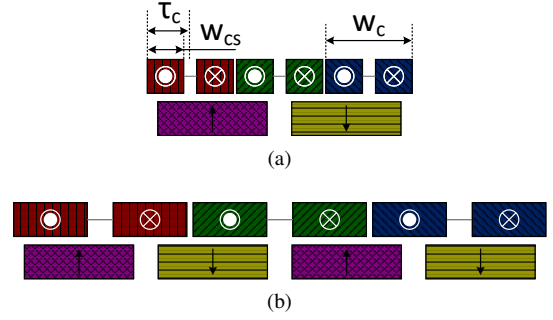


Fig. 7. Schematic of the smallest analysis periodicity for (a) three-coil over two-pole arrangement and (b) three-coil over four-pole arrangement.

TABLE I
WINDING FACTOR HARMONICS OF THE THREE-COIL CONCENTRATED CONFIGURATION FOR DIFFERENT K VALUES, WHERE THE 1ST AND 2ND HARMONICS CORRESPOND TO THE FUNDAMENTALS OF THE TWO-POLE AND FOUR-POLE CONFIGURATIONS, RESPECTIVELY.

ν	$K = 0.25$	$K = 0.50$	$K = 0.75$	$K = 1.00$
1	0.79	0.70	0.59	0.48
2	0.96	0.96	0.87	0.72
4	0.48	0.00	0.32	0.36
5	0.92	0.52	0.06	0.10
7	0.11	0.37	0.14	0.07

III. WINDING CONFIGURATIONS AND FACTORS

This section introduces typical winding configurations used in coreless stator machines and provides instructions for calculating the corresponding winding factors using the formulations developed in the previous section.

A. Single-layer Winding with the Number of Concentrated Coils Being in a Ratio of 3:2 or 3:4 with the Number of Poles

In this winding configuration, three concentrated coils, one per phase, form the smallest circumferential periodicity. This arrangement is compatible with either a two-pole or four-pole rotor. A 2-D representation of the winding layout is shown in Fig. 7. To calculate the winding distribution and pitch factors for this configuration, substitute $N_c = 3$ into (7) and (11). Since there is only one coil per phase within the smallest periodicity, the coil group factor is $K_g = 1$.

The winding factor for this configuration is calculated across different harmonic orders and coil-side-to-width ratios (K), and the results are summarized in Table I. The first harmonic corresponds to the fundamental component of the three-coil/two-pole configuration, and the second harmonic corresponds to the fundamental component of the three-coil/four-pole configuration.

B. Single-layer Winding with the Total Number of Concentrated Coils Being Equal to the Number of Poles ± 2

In this winding configuration, twelve coils form the smallest positive periodicity. In the first half of the pattern, two coils of the same phase with opposite winding directions are placed adjacent to each other, and in the second half, the same pattern

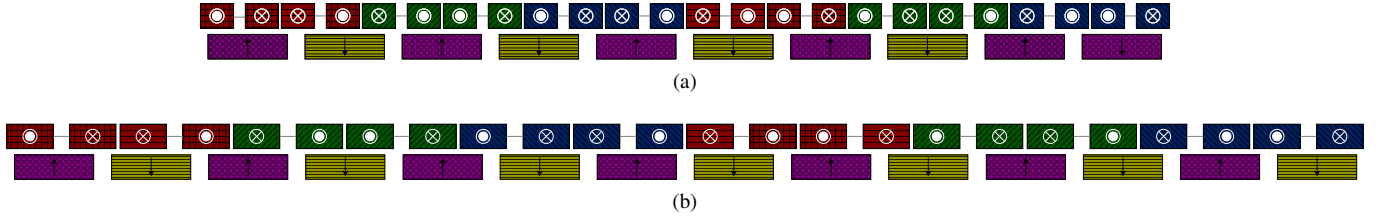


Fig. 8. Twelve-coil concentrated winding configuration that can operate with (a) ten poles and (b) fourteen poles. Adjacent coils of the same phase are wound in opposite directions. Due to periodicity, the last six coils repeat the first six in opposite winding directions, a feature that can be exploited to simplify FEA.

TABLE II

WINDING FACTORS OF THE TWELVE-COIL CONFIGURATION FOR VARIOUS K VALUES, WITH THE 5TH AND 7TH HARMONICS REPRESENTING THE FUNDAMENTALS OF THE TEN-POLE AND FOURTEEN-POLE CASES.

ν	$K = 0.25$	$K = 0.50$	$K = 0.75$	$K = 1.00$
1	0.06	0.05	0.04	0.03
5	0.88	0.79	0.68	0.55
7	0.96	0.92	0.81	0.66
11	0.15	0.20	0.21	0.18
13	0.04	0.13	0.17	0.15

TABLE III

WINDING FACTORS OF THE NINE-COIL CONFIGURATION FOR DIFFERENT K VALUES, WHERE THE 4TH AND 5TH HARMONICS CORRESPOND TO THE FUNDAMENTALS OF THE EIGHT-POLE AND TEN-POLE CASES.

ν	$K = 0.25$	$K = 0.50$	$K = 0.75$	$K = 1.00$
1	0.05	0.05	0.04	0.03
2	0.13	0.11	0.09	0.07
4	0.90	0.81	0.70	0.57
5	0.95	0.90	0.79	0.65
7	0.18	0.20	0.19	0.16

is repeated with the winding directions reversed. This arrangement is compatible with ten-pole or fourteen-pole rotors, and its 2-D representation is shown in Fig. 8. The distribution and pitch factors can be calculated by substituting $N_c = 12$ in (7) and (11).

For the twelve-coil winding, there are four coils associated with each phase within the smallest positive periodicity, and the coils are separated in phase by $\frac{\nu\pi}{6}$ radians on the ν^{th} harmonic. Thus, the coil-group factor can be written as:

$$K_g^{N_c=12} = \frac{1}{4} \left(e^{-j0} - e^{-j\frac{\pi\nu}{6}} - e^{-j\pi\nu} + e^{-j\frac{7\pi\nu}{6}} \right). \quad (12)$$

When ν is even, K_g becomes zero, and for an odd value of ν , it is equal to:

$$K_g^{N_c=12} = \sin\left(\frac{\pi\nu}{12}\right). \quad (13)$$

The winding factor for this configuration is calculated for various harmonic orders and K values, and the results are provided in Table II. The 5th and 7th harmonics correspond to the fundamental components of the 12-coil/10-pole and 12-coil/14-pole configurations, respectively.

C. Single-layer Winding with the Total Number of Concentrated Coils Being Equal to the Number of Poles ± 1

In this configuration, the fundamental periodic segment of the winding consists of nine coils. Within a single periodicity, three coils belonging to the same phase are positioned consecutively, with alternating winding directions. This topology can operate with either eight or ten poles, and a corresponding 2-D schematic is provided in Fig. 9. The distribution and pitch factors are obtained by setting $N_c = 9$ in (7) and (11).

In this nine-coil arrangement, each phase occupies three coils within one periodicity, and the phase displacement between adjacent coils is $\frac{2\nu\pi}{9}$ radians for the ν^{th} harmonic. Consequently, the coil-group factor is expressed as:

$$K_g^{N_c=9} = \frac{1}{3} \left(e^{-j0} - e^{-j\frac{2\pi\nu}{9}} + e^{-j\frac{4\pi\nu}{9}} \right), \quad (14)$$

which is equal to:

$$K_g^{N_c=9} = \frac{1}{3} \left(2 \cos\left(\frac{2\pi\nu}{9}\right) - 1 \right). \quad (15)$$

The winding factor of this topology is calculated across multiple harmonic orders and K values, with the resulting values summarized in Table III. In this case, the 4th and 5th spatial

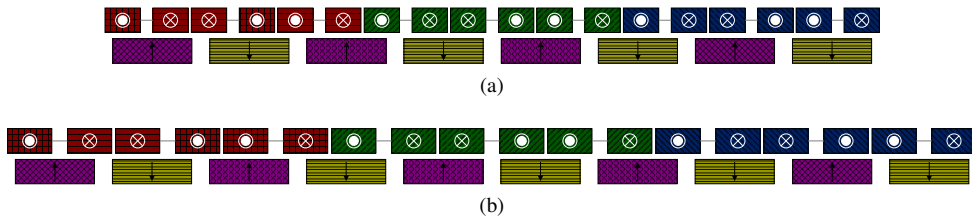


Fig. 9. Nine-coil concentrated winding configuration that can work with (a) eight poles and (b) ten poles. Adjacent coils of the same phase are wound in opposite directions.

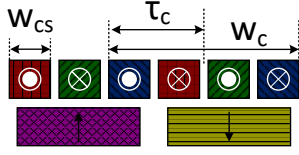


Fig. 10. Schematic of the smallest analysis periodicity for a fully pitched overlapping winding.

TABLE IV

CALCULATED WINDING FACTOR HARMONICS FOR DIFFERENT COIL-SIDE TO COIL-PITCH RATIOS (K), CORRESPONDING TO VARIOUS DIAMETERS IN A SINGLE-LAYER FULLY PITCHED OVERLAPPING WINDING. THE MAXIMUM K VALUE IN THIS WINDING TYPE IS 0.25.

ν	$K = 0.10$	$K = 0.15$	$K = 0.20$	$K = 0.25$
1	0.84	0.82	0.81	0.79
2	0.91	0.93	0.95	0.96
4	0.74	0.67	0.58	0.49
5	0.96	0.99	0.99	0.97
7	0.63	0.47	0.30	0.13

harmonics represent the fundamental components for the 12-coil/8-pole and 12-coil/10-pole arrangements, respectively.

D. Single-layer Fully Pitched Winding with Overlapping Ends

This configuration resembles a traditional fully-pitched distributed winding used in slotted machines, with overlapping coil ends. A 2-D schematic of the layout is provided in Fig. 10. The corresponding distribution and pitch factors are calculated by substituting $N_c = 3$ into (7) and (11). The resulting winding factors for different harmonic orders and K values are computed and presented in Table IV.

E. Three-Layer Equivalent Fully Pitched Windings with Concentrated or Wave Coils

This topology employs a three-layer winding structure, where each layer corresponds to the coils of a single phase. The number of coils per phase (i.e., per layer) equals the number of poles, resulting in a total coil count of three times the pole number. The layers are angularly displaced by 120° to realize a three-phase winding system.

A 2-D schematic of the coil arrangement is provided in Fig. 11. In this configuration, adjacent coils are wound in opposite directions, and their combined effect is equivalent to that of a fully pitched coil. A wave-winding variant can also be implemented to reduce copper losses by merging adjacent coil sides and alternately eliminating the end coils at the inner and outer radii.

This configuration corresponds to a 2-coil/2-pole periodic structure. Therefore, the winding distribution and pitch factors are obtained by setting $N_c = 2$ in (7) and (11). Each phase includes two coils within one electrical periodicity, and the phase shift between adjacent coils is $\nu\pi$ radians for the ν^{th} harmonic. Accordingly, the coil-group factor is given by:

$$K_g^{N_c=2} = (e^{-j0} - e^{-j\pi\nu}) = 1 - \cos(\pi\nu). \quad (16)$$

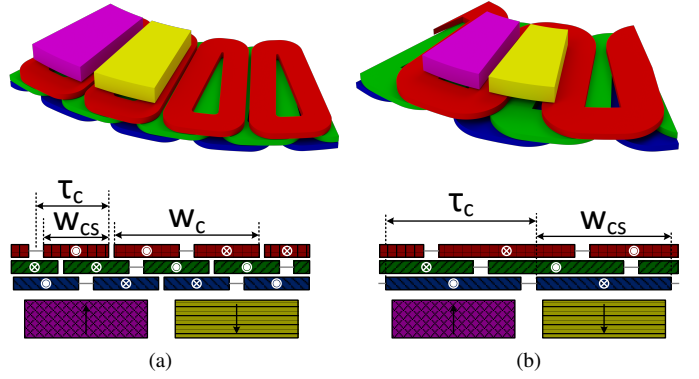


Fig. 11. Three-layer winding configuration, with each layer shifted by 120° relative to the others. Shown are the 3D view and their equivalent 2D representations for (a) concentrated-coil and (b) wave-winding types.

TABLE V

CALCULATED WINDING FACTORS FOR SINGLE-LAYER PER PHASE WINDINGS AT DIFFERENT K VALUES. THE EVEN HARMONICS ARE ZERO FOR THIS WINDING TYPE.

ν	$K = 0.25$	$K = 0.50$	$K = 0.75$	$K = 1.00$
1	0.97	0.90	0.78	0.64
5	0.47	0.18	0.07	0.13
7	0.14	0.13	0.11	0.09

In this winding configuration, all even harmonics are canceled out, and only odd harmonic components remain. The winding factor for various K values is computed and summarized in Table V.

IV. MAGNETIC FIELD, TORQUE, AND BACK-EMF CALCULATIONS

This section presents analytical modeling and formulations of the no-load magnetic flux density in the air-gap and electromagnetic torque calculation based on the Lorentz force theory, surface current density, and magnetic flux density distributions. Also, the back-EMF equation is derived from Faraday's law. The developed equations are based on equivalent two-dimensional modeling of axial-flux machines and are general to every coil/pole configuration.

For one pole pair of a double-sided, surface-mounted PM rotor, illustrated in Fig. 12a, analytical expressions for the no-load air-gap flux density have been reported in the literature [10], [36], [37]. Based on the generalized analytical formulation developed in [32], the normal and tangential air-gap flux density distributions can be expressed as follows:

$$B_x = \sum_{\nu=0}^{\infty} B_{n_{SPM}} \sinh\left(\frac{n\pi y}{\tau_p}\right) \sin\left(\frac{n\pi(2x + \tau_p)}{2\tau_p}\right) \quad (17)$$

$$B_y = \sum_{\nu=0}^{\infty} B_{n_{SPM}} \cosh\left(\frac{n\pi y}{\tau_p}\right) \cos\left(\frac{n\pi(2x - \tau_p)}{2\tau_p}\right), \quad (18)$$

where $n = 1 + 2i$, i is the harmonic order, and τ_p is the pole pitch at any diameter. The pole pitch can be calculated using

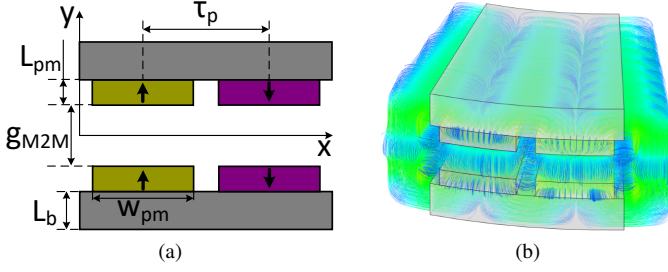


Fig. 12. Equivalent 2D model of a double-sided SPM rotor in an axial flux machine at an arbitrary diameter, showing the geometric variables (a), and 3D flux density distribution from FEA with matching boundary conditions (b).

$\tau_p = \frac{\pi D}{p}$, where D is the diameter and p is the pole number. $B_{n_{SPM}}$ can be obtained from:

$$B_{n_{SPM}} = 4B_r \cdot \frac{\sin(n\pi w_{PM}/2\tau_p)}{n\pi} \cdot \frac{\sinh(n\pi L_{PM}/\tau_p)}{\sinh(n\pi(L_{PM} + g_{M2M}/2)/\tau_p)}, \quad (19)$$

where B_r is the remanence of the PMs, w_{PM} is the PM width, L_{PM} is the PM length, and g_{M2M} is the magnet-to-magnet gap distance. It should be noted that the analytical formulation neglects magnetic saturation in the rotor back iron and assumes the back-iron material's relative permeability is sufficiently high compared to air.

The flux-density equation neglects edge effects from fringing flux at the inner and outer radii, as illustrated by the 3D FEA results in Fig. 12b. In coreless AFPM machines, the stator outer diameter is typically designed larger than the rotor outer diameter to partially utilize this fringing flux in torque production. The formulation also neglects rotor back-iron saturation, which can be evaluated in the final design stages using 3D FEA, as demonstrated in [38]. These assumptions are consistent with the objective of this work to provide a fast analytical tool for initial design and parameter selection rather than to replace detailed 3D FEA analysis.

To evaluate the accuracy of the magnetic flux-density expressions, a comparison was performed between the analytical model and 2D FEA. The model corresponds to the 36-pole coreless AFPM prototype design, described in the next section, with an outer diameter of 300 mm, an inner diameter of 210 mm, a magnet duty ratio of 0.8, and a M2M air-gap of 8.6 mm. The magnetic flux-density distribution obtained from magnetostatic 3D FEA with periodic boundary conditions for one pole pair of the motor is presented in Fig. 12b.

The normal and tangential components of the magnetic flux density computed at an axial position of $y = 3g_{M2M}/8$ are shown in Fig. 13. The analytical model demonstrates strong agreement with the FEA results, accurately capturing the harmonic contents across various rotor positions (x). At $y = 0$, the tangential flux density component is zero, and the normal component reaches its minimum value because of the double-sided structure.

In coreless stator machines, force generation derives from

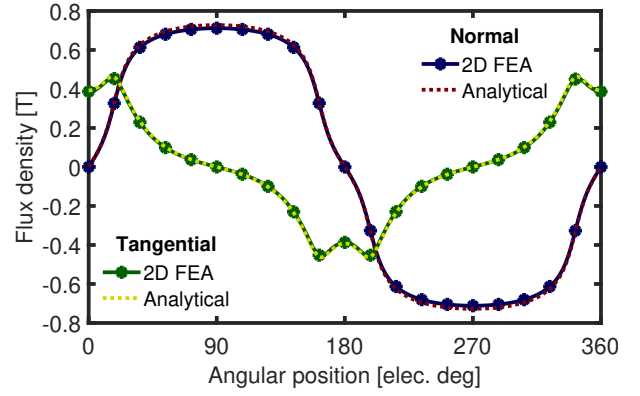


Fig. 13. Air-gap no-load flux density at $y = 3g_{M2M}/8$, showing the normal and tangential components calculated via 2D FEA and analytical tools.

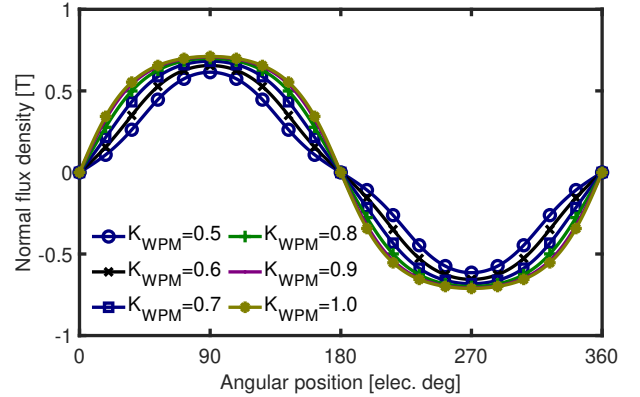


Fig. 14. Analytical no-load normal flux density curves at the air-gap midpoint for different PM spans.

the interaction between the magnetic flux density of the permanent magnets, B , and the surface current density, J . This interaction, known as the Lorentz force, can be described by [39]:

$$\vec{F} = \int_V \vec{J} \times \vec{B} dV, \quad (20)$$

where B can be calculated using (17), (18), and (19), and J can be derived from:

$$J = \frac{N_t I_p k_w}{\tau_c}, \quad (21)$$

where the number of turns per coil is represented by N_t , the peak phase current by I_p , k_w signifies the winding factor, and τ_c refers to the coil pitch.

In a coreless stator axial flux machine, the current density vector J is oriented in the radial direction, and the magnetic flux density B comprises two components: normal and tangential. The interaction between the normal component of the magnetic flux density (axial direction) and the radial current density generates a circumferential force, which produces torque. The interaction between the tangential component of the magnetic flux density and the radial current density generates a normal force acting between the stator and rotor.

It should be noted that the surface current density harmonics

TABLE VI
TYPICAL POLE/COIL COMBINATIONS IN CORELESS STATOR AFPM
MACHINES.

Poles (P)	Coils (N_c)	Coil Grp. No.	Phase shift γ ($^\circ$ e)	Periodicity frac. coeff. (K_M)
2	2 (3-L)	2	120 $^\circ$	$\overline{P}/2$
2	3 (Dis.)	2	120 $^\circ$	$\overline{P}/2$
2	3	1	120 $^\circ$	$\overline{P}/2$
4	3	1	240 $^\circ$	$\overline{P}/4$
10	12	2	120 $^\circ$	$\overline{P}/10$
14	12	2	240 $^\circ$	$\overline{P}/14$
8	9	3	120 $^\circ$	$\overline{P}/8$
10	9	3	240 $^\circ$	$\overline{P}/10$

follow the harmonic content of the winding factor, resulting in multiple pole harmonics. In the stator reference frame, the fundamental component rotates at synchronous speed, and the higher-order harmonics rotate at $1/\nu$ of the synchronous speed. In contrast, all harmonic components of the rotor magnetic flux density rotate at the synchronous speed. Torque is generated by the interaction of the rotor's magnetic flux density, including its harmonic components, and the fundamental component of the surface current density.

To further illustrate the influence of rotor magnetic flux density harmonics, the normal component of the flux density at the air-gap midpoint is plotted in Fig. 14 for different PM widths, where $K_{w_{pm}}$ denotes the PM-width-to-pole-pitch ratio. As the PM width increases, not only does the fundamental component of the flux density grow, but the third harmonic also increases. Because the third harmonic rotates at synchronous speed, its interaction with the fundamental component of the surface current density produces torque ripple. It is also observed that for $K_{w_{pm}} > 0.8$, the increase in flux density becomes negligible, indicating that a PM duty ratio of approximately 0.8 provides an optimal balance between magnet mass and torque production.

Substituting (18) and (21) into (20), the instantaneous electromagnetic torque expression can be derived as:

$$T = \frac{1}{6} K_M N_c N_t I_p K_D D_o^2 (1 - \lambda) \sum_{j=0}^2 \sum_{\nu=1}^{\infty} \sum_{i=0}^{\infty} \sin\left(\frac{\pi x}{\tau_p} + j\gamma\right) K_w B_{n_{SPM}}, \quad (22)$$

and the average torque, considering the fundamental components of surface current density and rotor magnetic flux density, can be calculated as:

$$T_{avg} = \frac{1}{2} K_M N_c N_t I_p K_{w1} K_D D_o^2 (1 - \lambda) B_1, \quad (23)$$

where K_M is the periodicity boundary condition coefficient, representing the number of repetitions of the base periodic segment, that varies for each coil/pole combination and is presented in Table VI; P and N_c are the pole and coil numbers of the base pole/coil combination; coil group denotes adjacent

coils of the same phase coil group denotes adjacent coils of the same phase within one periodicity; γ is the electrical phase difference between phases; $\overline{P}$ is the total pole number in 360 $^\circ$ mechanical; N_t is the number of turns per coil; I_p is the peak value of the phase current; K_{w1} is the fundamental component of the winding factor; K_D is a factor that take into account the diameter at which the calculation is performed and can be obtained from:

$$K_D = \overline{K_D} + (1 - \overline{K_D}) \lambda, \quad (24)$$

where $\overline{K_D}$ is the normalized diameter ratio to outer diameter (e.g., $\overline{K_D} = 0.5$ at the average diameter); D_o is the rotor outer diameter; λ is the rotor inner-to-outer diameter ratio; B_1 is the fundamental component of flux density ($i = 0$); and $B_{n_{SPM}}$ is the amplitude of the normal flux-density harmonics, can be calculated from (19) at $y = 0$ and scaled to a different position y by the factor of $\cosh\left(\frac{n\pi y}{\tau_p}\right)$.

In (22) and (23), the torque is shown to scale with the square of the machine diameter. It should be noted that the winding factor, K_w , itself contains a diameter-dependent term, as evident from the conductor distribution factor derived in (6). By substituting (6) into (23), the expression for the average torque can be rewritten as follows:

$$T_{avg} = \frac{1}{2w_{cs}} K_M K_{P1} K_{g1} N_c N_t I_p (1 - \lambda) B_1 K_D^2 D_o^3 \sin\left(\frac{w_{cs}}{K_D D_o}\right), \quad (25)$$

where the torque is proportional to the cube of the diameter, which is consistent with what has been shown for axial flux machines in the literature. In practice, torque can be evaluated at the average diameter, which provides a reasonably accurate approximation. For machines with relatively large outer-to-inner diameter ratios, the error caused by curvature effects can be reduced by evaluating torque at multiple diameters, multiple K_D , and averaging the results.

The back-EMF can be obtained from Faraday's law, which states that the induced voltage is equal to the time derivative of the flux linkage. By applying Faraday's law and assuming that all coils are connected in series, the resulting back-EMF is given by:

$$e = \frac{d\Phi}{dt} = \frac{K K_M N_t N_c P K_{w1} D_o (1 - \lambda^2) \omega_m}{K_D} \sum_{\nu=0}^{\infty} n B_{n_{SPM}} \cosh\left(\frac{n\pi y}{\tau_p}\right) \sin\left(\frac{n\pi (2x - \tau_p)}{2\tau_p}\right), \quad (26)$$

where K is a constant equal to $\pi^2/180$, P is the pole number within smallest periodicity, and ω_m is the rotor speed in rpm.

It should be noted that the proposed winding-factor formulation and the derived torque expressions are applicable to coreless-stator AFPM machines with Halbach-array rotors. The only required modification is to replace the surface-mounted PM air-gap flux density expressions with the cor-

responding Halbach array flux density equations. Analytical expressions for the normal and tangential components of the air-gap flux density produced by Halbach array PM rotors are presented and discussed in detail in [1], [32].

V. DISCUSSION AND DESIGN CONSIDERATIONS

The early stages of the design process include the selection of pole count and coil/pole combinations. The amplitudes of the harmonic winding factors, which are exemplified in Fig. 15 with a typical value of $K = 0.6$, are important together with other considerations, such as wire type (Litz or PCB), loss mitigation, design flexibility, and fault tolerance. The coil/pole combinations provide degrees of freedom in selecting, for a given outer diameter, the pole number, which must be an integer multiple of the periodicity listed in Table VI. For smaller diameters, combinations with lower periodicity (see the last column K_M in the table) allow finer pole-count adjustments, whereas for larger diameters, higher pole numbers enable a wider range of feasible coil/pole combinations.

In coreless stator AFPM machines, the main electromagnetic loss components are the Joule losses, together with the conductor and the PM eddy-current losses, respectively. The choice of coil/pole combination influences these losses and the selection of corresponding mitigation strategies. Recent papers discussing such aspects include, for example [1], [13].

Traditionally, coreless stators used Litz wire [22] and more recently, especially for high-volume cost-competitive applications, PCB has been employed [15]. In particular, manufacturing constraints arise from bending Litz wires to produce coils, which may limit the applicability for high pole count designs suitable for operation with power converters with wide band gap (WBG) devices operating at high fundamental and switching frequency [40]. Such limitations are less restrictive for PCB stators for which the equivalent coil geometry may be manufactured with greater flexibility, as further discussed in the following.

In PCB stator windings, thin conductors and multiple parallel paths can be implemented using multilayer technology. Conductor transposition between layers is also possible to reduce eddy-current losses and minimize circulating current. The strategies for mitigating eddy-current and Joule losses in PCB stator windings, including layer transposition, vias, and interconnections, have been previously reported by our extended research group [41].

In certain coil/pole combinations, the circumferential distance between coils of the same phase becomes large, such as in twelve-coil and nine-coil concentrated coil configurations. In these cases, four or six coils of the other phases are positioned between adjacent coils of the same phase, making the required via layout more complex or enlarging the end-winding region and PCB diameter.

The fully pitched distributed winding presents challenges for inter-layer connections due to overlapping end coils, which reduce the available space for routing in a PCB. A three-coil concentrated winding configuration offers more feasibility in

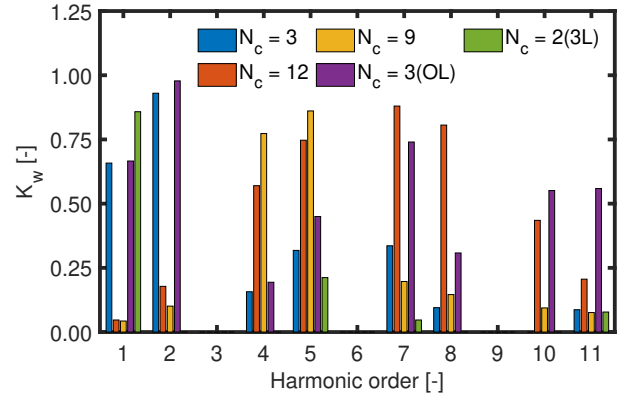


Fig. 15. Winding-factor harmonic distributions for typical winding configurations in coreless stator AFPM machines at $K = 0.6$. The label 3 (OL) denotes a fully pitched overlapping winding, and 2 (3L) represents a three-layer equivalent fully pitched winding configuration.

this regard, particularly when a three-coil/two-pole arrangement is used instead of a three-coil/four-pole configuration. This is because, for the same pole number, the three-coil/two-pole design provides twice as many coils, enabling twice the degree of freedom for each coil parallel paths and conductor transposition. The drawback of the three-coil/two-pole configuration is its relatively poor winding factor.

The three-layer concentrated-coil winding enables straightforward routing between layers in a PCB because the coils of the same phase are positioned adjacent to one another, which is a significant advantage. This leads to a simpler PCB layout and provides greater freedom in selecting conductor size and the number of parallel paths, potentially improving eddy- and circulating-current loss mitigation while maintaining a reasonable winding factor.

Depending on the machine's rated power and voltage levels, insulation between coils of different phases must be carefully considered to prevent short-circuit faults. In coil/pole combinations where all phases are placed on a single PCB, the required inter-phase insulation may reduce the equivalent slot fill factor. In this context, a three-layer PCB stator winding configuration offers higher reliability and potentially a greater equivalent slot fill factor than other configurations.

As loss mitigation in PCB stators is critical, effective cooling is also essential. Thermal management in coreless stator AFPM machines is generally simpler than in conventional cored machines due to improved winding accessibility [12]. In conventional slotted machines, windings are embedded in stator slots, limiting direct cooling mainly to the end windings. In contrast, coreless configurations allow airflow to reach a larger portion of the winding surface, enabling greater heat removal and improving thermal performance.

Cooling in PCB stator machines requires more detailed thermal modeling due to the multilayer PCB structure and the temperature gradients across FR4 substrates and copper traces. As shown in [42], by maintaining reasonable current density and properly utilizing both airflow convection on the PCB stator and conductive heat transfer through effective

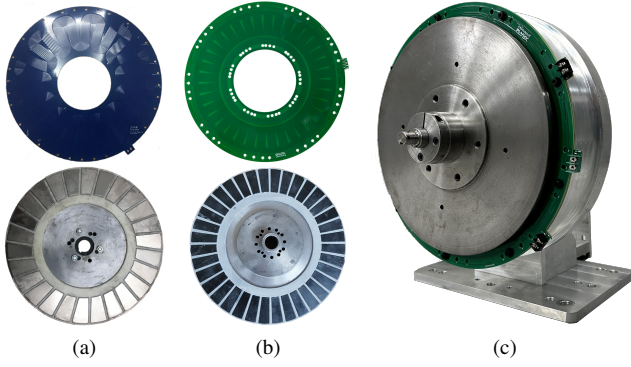


Fig. 16. Coreless PCB-stator AFPM machine prototypes with three-layer winding, (a) 26-pole rotor with wave winding, (b) 36-pole rotor with concentric winding, and (c) assembled prototype.

thermal contact with the motor frame, PCB stators can achieve thermal performance comparable to epoxy-immersed Litz-wire windings. Integrating drive circuitry on the same PCB as the stator winding introduces additional thermal challenges [43], requiring more comprehensive thermal analysis.

The final loss component is PM eddy-current loss due to stator field harmonics. These losses can be mitigated by suppressing unwanted harmonics at the source. For example, in a concentrated three-coil winding operating with the second harmonic (coil/pole = 3/4), the first harmonic does not contribute to torque and induces PM eddy currents. This harmonic can be canceled using a two-layer winding with an appropriate coil offset, preserving the torque-producing harmonic while reducing PM eddy-current losses [44].

VI. EXPERIMENTAL AND FEA VALIDATION

This section aims to compare and validate the accuracy of the developed analytical model against 3D FEA results and experimental measurements. Several considerations are essential for ensuring a fair comparison. The analytical model neglects rotor back-iron saturation; therefore, the back-iron thickness must be chosen such that the flux density in both the 3D FEA model and the prototype remains below the material's knee point. In the equivalent 2D representation, the motor's radial length is equivalent to the stack length, which is the active length of the coil side (the coil radial length minus twice the coil side width). In this study, torque and back-EMF are evaluated at the average diameter. They may also be computed using a multi-slice approach, sampling multiple radial positions and averaging the results, to achieve a potentially higher accuracy.

Two different coreless AFPM machine prototypes were used to verify the analytical model. These prototypes were developed at different times and have different ratings and pole numbers, yet both share an outer diameter of 300 mm and employ PCB stators with a three-layer winding configuration, which is a practical option for PCB windings as reviewed in the previous section.

TABLE VII
SPECIFICATIONS AND MAIN DIMENSIONS OF THE PROTOTYPED CORELESS-STATOR AFPM MACHINES.

Parameter	Prot. #1	Prot. #2	Unit
Rotor outer diameter, D_{ro}	270	305	mm
Rotor inner diameter, D_{ri}	182	208	mm
Stator outer diameter, D_{so}	305	310	mm
Stator inner diameter, D_{si}	145	200	mm
Pole number, P	26	36	–
Winding type	3L wave	3L concentric	–
PM axial length, L_{pm}	5	5	mm
PM arc-to-pole pitch ratio, $k_{w_{pm}}$	0.8	0.8	–
PCB axial length per phase, L_{pcb}	2	2	mm
Turns in series per phase	182	162	–
Mechanical airgap, g	1.5	1.0	mm
Rated torque	14	19	Nm
Rated current	12	12	A
Phase resistance	0.6	0.6	Ω

One prototype employs a three-layer concentrated coil configuration with 36 poles, whereas the other uses a three-layer wave winding with 26 poles. The purpose of this section is to validate the proposed analytical model rather than compare the performance of the two motors. Using two distinct prototypes allows for demonstrating analytical model accuracy across different winding layouts and pole counts.

The rated torque of both prototypes was evaluated using the proposed analytical model, 3D FEA, and experimental measurements. For the 36-pole prototype, the analytical model exhibited deviations of 1.6% and 3.3% compared to the 3D FEA and experimental results, respectively. For the 26-pole prototype, the corresponding errors were below 1.0% and 4.0%. These results demonstrate satisfactory agreement among the analytical predictions, 3D FEA simulations, and experimental measurements, confirming the accuracy and reliability of the proposed model.

The back-EMF of both prototypes was also evaluated at 1,000 rpm using (26) and compared with 3D FEA and experimental measurements. The per-phase back-EMF waveforms over one electrical cycle, shown in Fig. 17, demonstrate strong agreement among the calculations and measurements.

It should be noted that in the three-layer winding configuration, the middle PCB is located farther from both rotor surfaces and therefore experiences a lower flux density and reduced B-EMF than the two outer PCBs. This discrepancy increases as the PCB thickness becomes larger. In the analytical model, the B-EMF and torque are computed at a distance from the rotor surface, represented by the parameter y in the flux-density expression. To increase accuracy, particularly in machines with a relatively large M2M gap, the torque can be evaluated at multiple y positions, such as the centerlines of the outer PCBs and the middle PCB. The overall torque can be obtained by averaging the torque values calculated at these positions.

In coreless stator AFPM machines, the d- and q-axis inductances are equal ($L_d = L_q$) due to the absence of

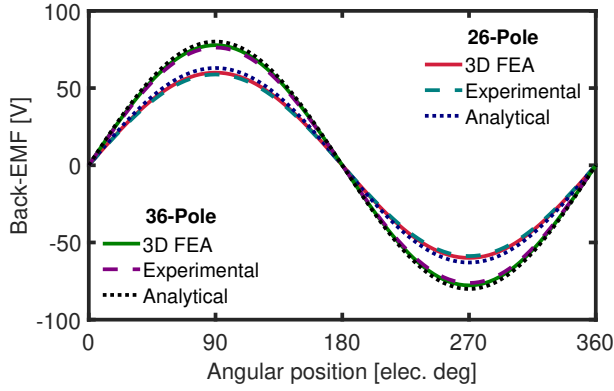


Fig. 17. Single-phase comparison of analytical, 3D FEA, and experimental back-EMF waveforms at 1,000 rpm.

TABLE VIII
COMPARISON OF ANALYTICAL CALCULATIONS, 3D FEA RESULTS, AND EXPERIMENTAL MEASUREMENTS OF TORQUE AND BACK-EMF FOR BOTH PROTOTYPES.

Prot.	B-EMF [V]			Torque [Nm]		
	Analy.	3D FEA	Exp.	Analy.	3D FEA	Exp.
Prot. #1	44.9	43.1	43.0	14.4	14.0	13.8
Prot. #2	80.1	77.8	77.1	19.3	19.0	18.9

magnetic saliency. Consequently, the maximum torque per ampere (MTPA) operating condition occurs at $i_d = 0$, corresponding to current injection aligned with the back-EMF. This is the operating condition used for all analytical and experimental results presented in the paper. Furthermore, the phase inductances of coreless AFPM machines are typically orders of magnitude smaller than those of conventional AFPM machines, generally in the range of few micro-Henries. Owing to these very small inductances, flux-weakening operation is therefore impractical in this class of machines.

In addition, because magnetic saturation is negligible in coreless stator AFPM topologies, the electromagnetic torque scales linearly with current amplitude over the practical operating range. The instantaneous torque expression provided in (22) can accommodate flux-weakening operation through appropriate adjustment of current magnitude and phase angle. Given the low inductance and non-salient nature of coreless AFPM machines, such operating conditions are not feasible, and experimental validation in a single operating condition at MTPA is sufficient to validate the calculations.

VII. CONCLUSION

The new theoretical formulations and analytical modeling framework introduced in this paper for coreless-stator AFPM machines provide essential guidance for the initial steps of the design process, including the selection of main variables, number of poles, coil/pole combination, and winding pattern. The generalized winding-factor theory proposed considers the conductor distribution, pitch, and coil-group factors, which are applicable to typical and most frequent coreless AFPM motor

topologies employing a double-sided PM rotor, and was exemplified for five different typical winding layouts. Successful validation within 5% error was performed against 3D FEA and experimental results from two prototype motors with different poles and winding types. The equations derived for no-load air-gap flux density, back-EMF, and torque, based on an equivalent 2D modeling method for axial-flux machines, are most useful for preliminary design and maybe complemented by 3D FEA only later on in the design process, resulting in substantial savings of computational resources and time.

The exemplified analysis indicated that proposed special 2-coil/2-pole distributed windings, and 2-coil/2-pole three-layer windings for coreless-stator AFPM machines have very high winding factors, comparable with those from winding patterns derived from more conventional concentrated coil configurations. The study was complemented by further design considerations, including the choice of stator conductor material (Litz wire or PCB), electromagnetic loss mitigation, manufacturability, fault tolerance, and reliability, indicating substantial advantages for the proposed windings.

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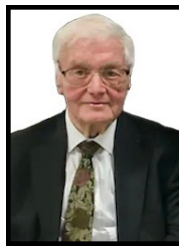


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