Two-Level Surrogate–Assisted Differential Evolution Multi–Objective Optimization of Electric Machines Using 3D FEA

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A two level surrogate assisted optimization algorithm is proposed for electric machine design using 3D finite element analysis (FEA). The algorithm achieves the optima with much fewer FEA evaluations than conventional methods. It is composed of interior and exterior levels. The exploration is performed mainly in the interior level which evaluates hundreds of designs employing affordable kriging models. Then, the most promising designs are evaluated in the exterior loop with expensive 3D FEA models. The sample pool is constructed in a self-adjustable and dynamic way. A hybrid stopping criterion is used to avoid unnecessary expensive function evaluations.

***Index Terms*—Optimization, 3D FEA, axial flux machines, surrogate model, kriging.**

I. Introduction

ELECTRIC machine design requires an optimization algorithm to achieve the best result. Conventional optimization algorithms often use thousands of design evaluations. Hence, 3D FEA is not affordable. However, 2D models are not accurate for machines with 3D flux paths, such as axial flux or transverse flux machines, or for studying skew angle, overhang, or end coils. One solution is to use surrogate models, although the accuracy of these can decline in a wide and nonlinear search space. Another solution utilizes algorithms with a minimum number of designs evaluations. A combination of these two solutions is proposed here.

Currently, for the optimal design of electric machines, population based evolutionary algorithms are widely used with the differential evolution (DE) method being a typical choice [1], [2]. According to DE, following initialization of a random population, offspring multiple successive generations are created by differential mutation, an operation achieved by adding a scaled difference of two previous designs to a third parent design. The resulting children will survive to the next generation if they achieved improvement for all multi-objective performance indices considered as part of the optimal design problem. Only a minimum number of control parameters, namely the scaling factor and crossover probability, are considered in the DE algorithm and the global optimum can be achieved regardless of the initial designs. Nevertheless, a major disadvantage of conventional DE is that it requires the evaluation of a very large number of generations and candidate designs, which for electrical machines are typically based on computational expensive FE models [3], [4], [5]. For example, a previous optimal design problem with five independent variables employed more than four thousand candidate designs [3], while using the novel algorithm proposed in the current paper this number can be reduced by one order of magnitude, to only a few hundreds.

Surrogate modeling is a suitable replacement for expensive measurements. The response surface methodology (RSM) is a popular example of surrogate model application in electric machine design; these mathematically estimate the performance of a design, and can be classified into global or local meta-models. A global surrogate model is a regression model built from a predetermined function, as is with RSM. Local surrogate interpolations are obtained from spatial functions passing through all sample points. The Guassian process, known as kriging, is an example of local surrogate models where a greater weight is put on closer sampled data points [6]. Kriging models gained popularity in geostatistics [7] and proved to be a practical estimation tool. Other fields, including electric machine design optimization, have taken advantage of this method.

Kriging-based optimization has emerged in the design of electromagnetic devices, and more recentely electric machines [8], [9], [10], [11]. This paper makes further contributions by a special two-level optimization algorithm which eliminates the estimation errors on significant designs. The algorithm has a dynamic sample pool with self-adjusting capabilities for problems with different levels of complexity, thus avoiding unnecessary evaluations. The proposed algorithm particularly saves significant time for multi-objective optimization problems, which have a higher level of complexity with several objectives, such as losses, mass, cost, etc., to be simultaneously considered.

The next section describes the kriging model. The proposed algorithm is explained in detail in section III. The algorithm is employed and compared with conventional approaches, for test functions with different numbers of variables and objectives in section IV. In section V the performance of the method is demonstrated with axial flux machines using 3D FEA models. A discussion and conclusions are finally presented.

TABLE I. The RMS estimation error of predictions made by 3 surrogate models, for magnet eddy losses of an example AFPM machine.

Surrogate model	2^{nd} order regression	Eureqa	kriging
RMS estimation error	48.5%	143.5%	13.5%

II. KRIGING SURROGATE MODELING

Kriging is a local fitting model that, unlike other methods, does not use one predetermined polynomial function to estimate every unsampled design. It has two components: trend and residual. The benefits of kriging models rise from the residual component that addresses the error between the actual data points and the general trend. There are different types of kriging modeling methodologies mostly categorized by the trend component [12]. The universal kriging model is the most complicated and accurate, and is used here. The kriging estimation methods can be expressed as

$$\hat{Y} = \hat{X}\beta + r^T R^{-1}(Y - X\beta) \ ; \qquad (1)$$

$$r_i = exp[-\Sigma_{t=1}^{k}\theta_t|\hat{x}_t - x_{i,t}|^2] \ ; \quad i,j = 1,...,n, \qquad (2)$$

$$R_{i,j} = exp[-\Sigma_{t=1}^{k}\theta_t|x_{i,t} - x_{j,t}|^2] \ ; \quad i,j = 1,...,n. \qquad (3)$$

where $\hat{Y}$ is the unsampled design to be predicted, based on known sample designs, i.e. X and Y; β is the regression coefficients that can be obtained using methods such as least squares; n is the number of sampled designs; and k the number of optmization variables. Kriging weights, r^T and R^{-1} are derived from a covariance function or semivariogram and a maximum likelihood estimation (MLE) [13].

In order to assess the perdition accuracy of kriging models versus global surrogate models, i.e., second order regression and Eureqa [14], [15], the magnet eddy losses of an example axial flux machine was studied. Fifty designs were evaluated using 3D FEA. The RMS estimation error for 15 unsampled designs were compared to FEA calculations. As presented in Table I, kriging estimations are more accurate than the others. Henceforth, predictions made by the kriging surrogate model will be referred to as inexpensive evaluations, and calculations made by the 3D FEA as expensive evaluations.

III. NOVEL OPTIMIZATION ALGORITHM

The proposed approach composes an exterior level evolutionary algorithm, replacing the mutation with an interior level complete DE optimization. After the first generation of the main loop (exterior), parents are, in fact, estimated Pareto (non-dominated) designs, hence, close to the actual Pareto front. The exterior loop uses expensive function evaluations for the estimated Pareto designs to correct the estimation errors made by the interior loop. The optimization algorithm flowchart is given in Fig. 1 and explained as follows.

A. Initial sample pool

To efficiently generate the sample pool, the number of designs and their locations need to be considered. Larger number of samples is ideal; however, for each sample, computational effort should be performed, so it is important to reduce the samples through effectively locating them.

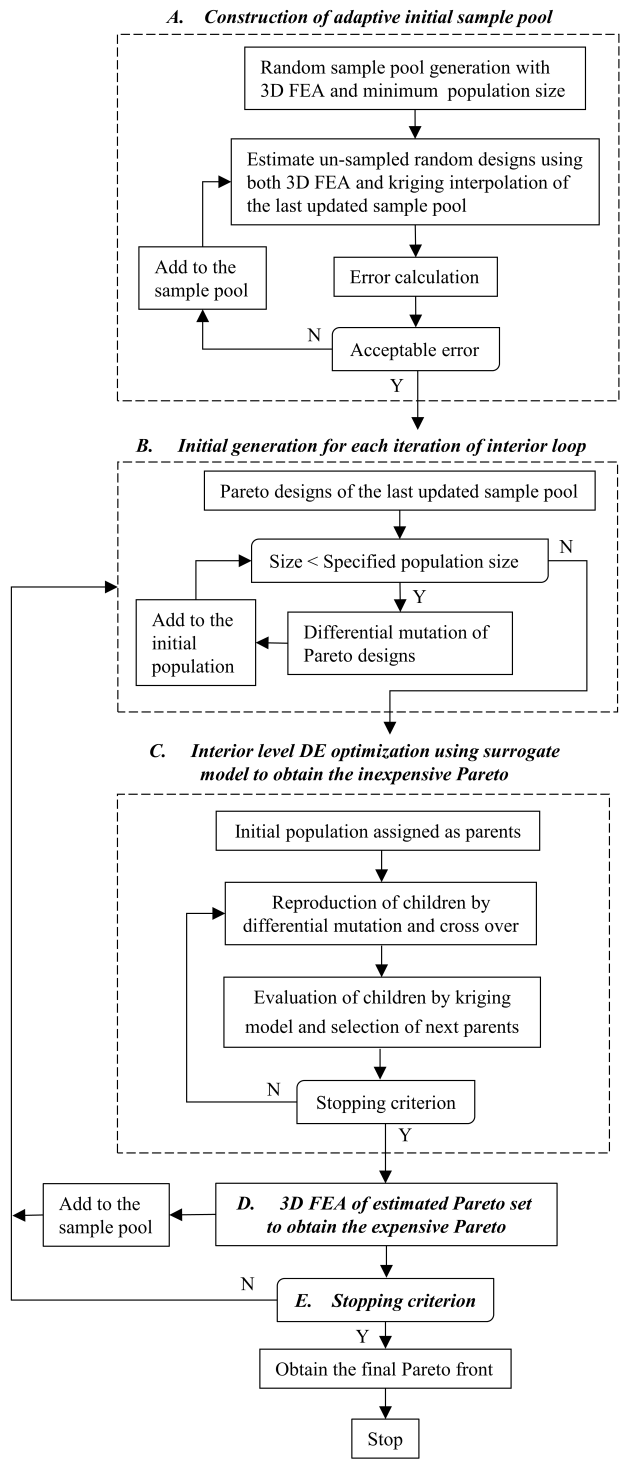


Fig. 1. The two-level optimization algorithm with an interior loop based on DE and kriging surrogate models.

Sampling strategies can be categorized in two groups: (1) space filling metods for exploration purposes, and (2) sequential infilling sampling that improves exploitation [16]. In local surrogate model based optimization, infilling samples are very important. In this algorithm, infilling is performed in two stages: in the initial sample construction and after each generation. After generating the minimum sample size, the performance of a limited number of unsampled designs, e.g.

10, is evaluated with the kriging model as well as simulated with FEA and an estimation error is calculated. If the error is larger than a pre-set limit, e.g. %5, these designs are added to the pool and a new batch of unsampled designs are evaluated. This process constantly increases the sample pool size until satisfactory estimations are achieved for all test designs. As each outer level generation ends, the sample pool size increases only in the promising parts of the search space.

It should be mentioned that Design of Experiment (DOE) can be utilized as a systematic approach to generate the initial sample pool. However, when the search space is large, designs span over a wide range and nonlinearity is expected, so a higher number of levels for acceptable resolution is required; hence, a large number of designs are required by DOE to achieve reasonable resolution.

B. Initial generation for each iteration of interior loop

The initial population of the interior DE optimization is the Pareto designs of the latest sample pool. If the number of Pareto designs is more than the population size, extra designs are randomly eliminated; if they are less, additional designs are obtained using differential mutation.

C. Interior level

The interior level is a conventional DE optimization using inexpensive function evaluations. The output of this block is an estimated Pareto.

D. 3D FEA of promising designs

It is expected to have some estimation error for the inexpensive Pareto front. To correct that, the Pareto designs are evaluated with 3D FEA and replaced with estimations.

E. Stopping criterion

Multi-objective optimizations usually set a maximum number of function evaluations or maximum number of generations as the termination criterion. For the algorithms that converge to the optima very fast, this criterion can cause many dispensable generations which is vital to avoid, in order to reduce expensive evaluations. Here, a hybrid stopping criterion is used. Negligible improvement in the tips and the middle point of the Pareto front, for a few consecutive generations, will satisfy the third stopping criterion. Meeting any of these criteria, stops the algorithm.

IV. Algorithm Implementation, Validation and Design Examples

The proposed optimization algorithm is implemented and validated using the test function DTLZ2 [17]. This function is capable of assessing the algorithm for problems with different levels of complexity, i.e., number of optimization variables and objectives. DTLZ2 functions with 1-3 objective and 4-12 variables were studied. Table II represents the average results of 5 runs for each scenario. Even for high number of variables, the proposed algorithm outperforms the conventional. For very complex problems with more than 3 objectives and 12 variables, the sample pool construction needs almost as many function calls as the total evaluations in conventional DE algorithm. Hence, their performances are comparable.

TABLE II. The results of the optimization algorithm presented in Fig. 1 for the test function (DTLZ2) with different number of objectives and variables. The results are the average of 5 runs for each scenario.

Variables	4		8		12	
Algorithm	DE	2L SA	DE	2L SA	DE	2L SA
1 objective; the population size of each generation is 5						
Generations	39.8	4	51	4	65.3	5
Func Evals	199	64	255	90	326.5	122.2
2 objectives; the population size of each generation is 10						
Generations	46.4	4	70.2	7 4.3	84.8	5.8
Func Evals	464	170	702	384.7	848	429.4
3 objectives; the population size of each generation is 15						
Generations	46.4	4	52.4	4	63.8	4
Func Evals	696	440	786	800	957	1060

TABLE III. Optimization variables of the conventional machine.

Variable	Unit	Minimum	Maximum
Air-gap	mm	1.8	5.0
Magnet thickness	mm	3.0	10.0
Tooth width / pitch		0.35	0.65
Pole arc / pitch		0.65	0.85
Statot yoke	mm	9	18
Rotor yoke	mm	5	11

TABLE IV. Optimization variables of the coreless machine.

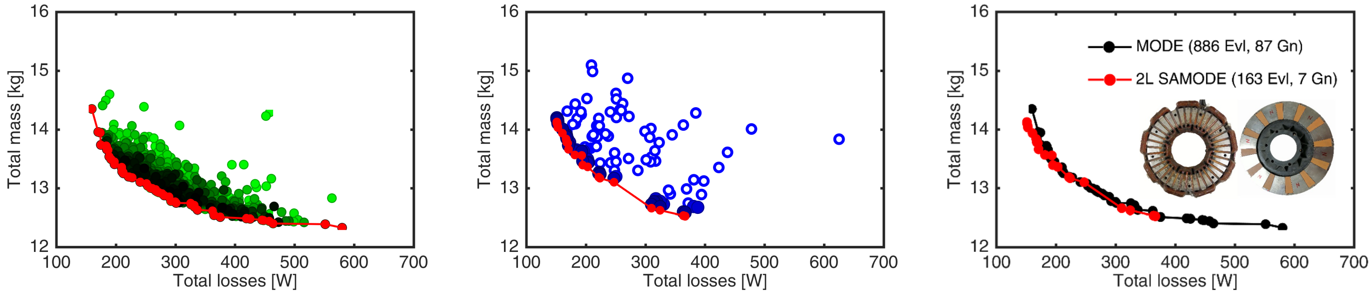
Variable	Unit	Minimum	Maximum
Air-gap	mm	0.5	2.0
Magnet thickness	mm	1.0	3.5
Pole arc / pitch		0.65	0.85
Rotor yoke	mm	4.0	8.0

Two implementations of the algorithm for axial flux permanent magnet (AFPM) machines with different numbers of variables were also studied: a commercially available conventional AFPM motor with single-rotor single-stator topology [18], and a multi-disc coreless machine with 2 stators [19]. The number of optimization variables are 4 and 6 for the coreless and conventional machines, respectively, represented in Table III and IV. The minimization of two objectives is simultaneously considered for the multi-objective design studies: active material mass and total losses at rated operation. All designs are evaluated at the rated torque and speed.

The optimization progress and Pareto front obtained from the conventional multi-objective DE (MODE) and the two-level surrogate assisted algorithm (2L SAMODE) are shown in Figs. 2 and 3. Darker colored designs evolved in more recent generations. The hollow data points represent designs in the initial sample pool.

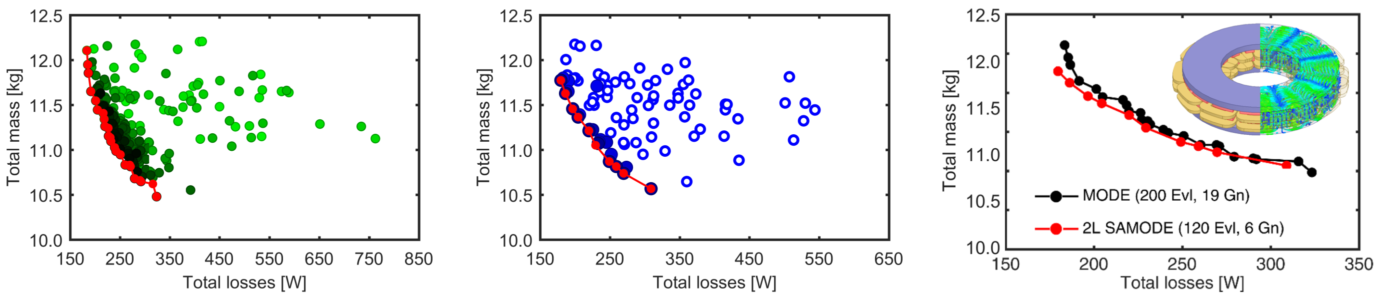
V. Discussion

Both MODE and 2L SAMODE have achieved relatively similar Pareto front, which proves them to be the true Pareto front. In the coreless machine case, the MODE algorithm has terminated prematurely and needs a more strict stopping criterion. The reason can be explained by considering that total number of function calls in two-level optimization is the sum of the interior loop (inexpensive) and the exterior loop (expensive) function evaluations. This sum will be orders



(a) (b) (c)

Fig. 2. The optimization results for the commercial motor [18] at the rated torque of 56 Nm, using (a) the conventional MODE, (b) the 2 level SAMODE, and (c) the Pareto fronts obtained from the two algorithms. The darker colors in (a) and (b) represent the designs evolved in more recent generations.



(a) (b) (c)

Fig. 3. The optimization results for a coreless AFPM motor at the rated torque of 28 Nm, using (a) the conventional MODE, (b) the 2 level SAMODE, and (c) the Pareto fronts obtained from the two algorithms. The darker colors in (a) and (b) represent the designs evolved in more recent generations.

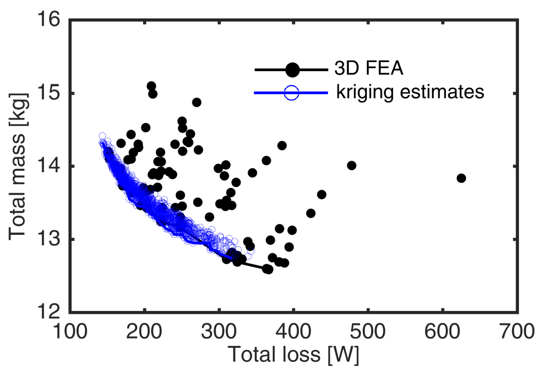


Fig. 4. Employing the surrogate model is estimating designs in order to fill in the gaps in Pareto front.

of magnitude more than the conventional algorithm, which improves exploration and exploitation; hence, it is likely that Pareto designs are achieved faster.

In case of the AFPM motor in Fig. 2c, the proposed 2L SAMODE achieves the Pareto front with only 163 3D FEAs, while conventional MODE needs 886 FEAs. For the coreless machine, Fig. 3c, the problem is less complicated, due to fewer number of variables and eliminated possible saturation in teeth. The Pareto front is obtained with 120 FEAs for 2L SAMODE, and 200 FEAs for MODE.

It is desired to include more designs in the Pareto front, in order to provide more alternatives in the design selection. The kriging model can be used to fill in the gaps. This is demonstrated in Fig. 4, using the 2L SAMODE results in Fig. 2b. First, the variables range is limited to the designs in the Pareto front. Then thousands of designs within that range are estimated using the kriging model. Relatively accurate estimations are expected, since the sample pool has a better resolution closer to the Pareto. In case of MODE, to have a more complete Pareto front, the population of each generation should be increased, which requires even more FEAs.

VI. Conclusion

A two-level surrogate-assisted DE based optimization is proposed for use with electric machine design problems with 3D FEA. The results show that the algorithm outperforms conventional methods as it requires substantially fewer design evaluations. The two-level evaluation of Pareto designs results in an efficient exploration and exploitation approach so that the global optima can be located within the first few generations, depending on the accuracy of the kriging model.

This algorithm, unlike most surrogate assisted optimizations, does not solely rely on estimated values; it has a dynamic sample pool that stops increasing in size once the estimation error is sufficiently small, and gradually improves the kriging model resolution only around Pareto designs. These make it possible to achieve accurate final results, avoid unnecessary expensive evaluations, and converge faster. The constructed kriging model can also be used for post processing purposes, such as developing a full Pareto front, and thereby allowing selection from a larger number of optimum designs.

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